MTH 111, Math for Architects, Final Exam, Spring 2014

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QUESTION 1. (i) Let $f(x) = -x^3 + 8x - 1$. The slope of the tangent line to the curve at the point $(1, 6)$

    a. 5

    b. 6

    c. 13

(ii) Let $f(x) = -2x^3 + 24x + 1$. Then $f(x)$ increases on the interval

    a. $x \in (-\infty, -2) \cup (2, \infty)$

    b. $x \in (-\infty, -\sqrt{12}) \cup (\sqrt{12}, \infty)$

    c. $x \in (-\sqrt{12}, \sqrt{12})$

    d. $x \in (-2, 2)$

(iii) let $f(x) = 3e^{(x^2 - x - 2)} + 4x + 5643217689$. Then $f'(2)$

    a. 13

    b. 11

    c. 9

    d. 7

(iv) Let $f(x) = (x + 1)e^{(x - 2)} + 3x + 14523$. Then

    a. $f(x)$ has an inflection point at $x = -3$

    b. $f(x)$ has an inflection point at $x = -2$

    c. $f(x)$ has no inflection points

    d. $f(x)$ has an inflection point at $x = -1$

    e. $f(x)$ has an inflection point at $x = 2$

(v) Let $f(x) = -x(2x - 32)^7 + 1550$. Then

    a. $f(x)$ has a local maximum at $x = 32/9$

    b. $f(x)$ has a local maximum $x = 2$

    c. $f(x)$ has a local minimum at $x = 32/9$

    d. $f(x)$ has a critical value when $x = -16$
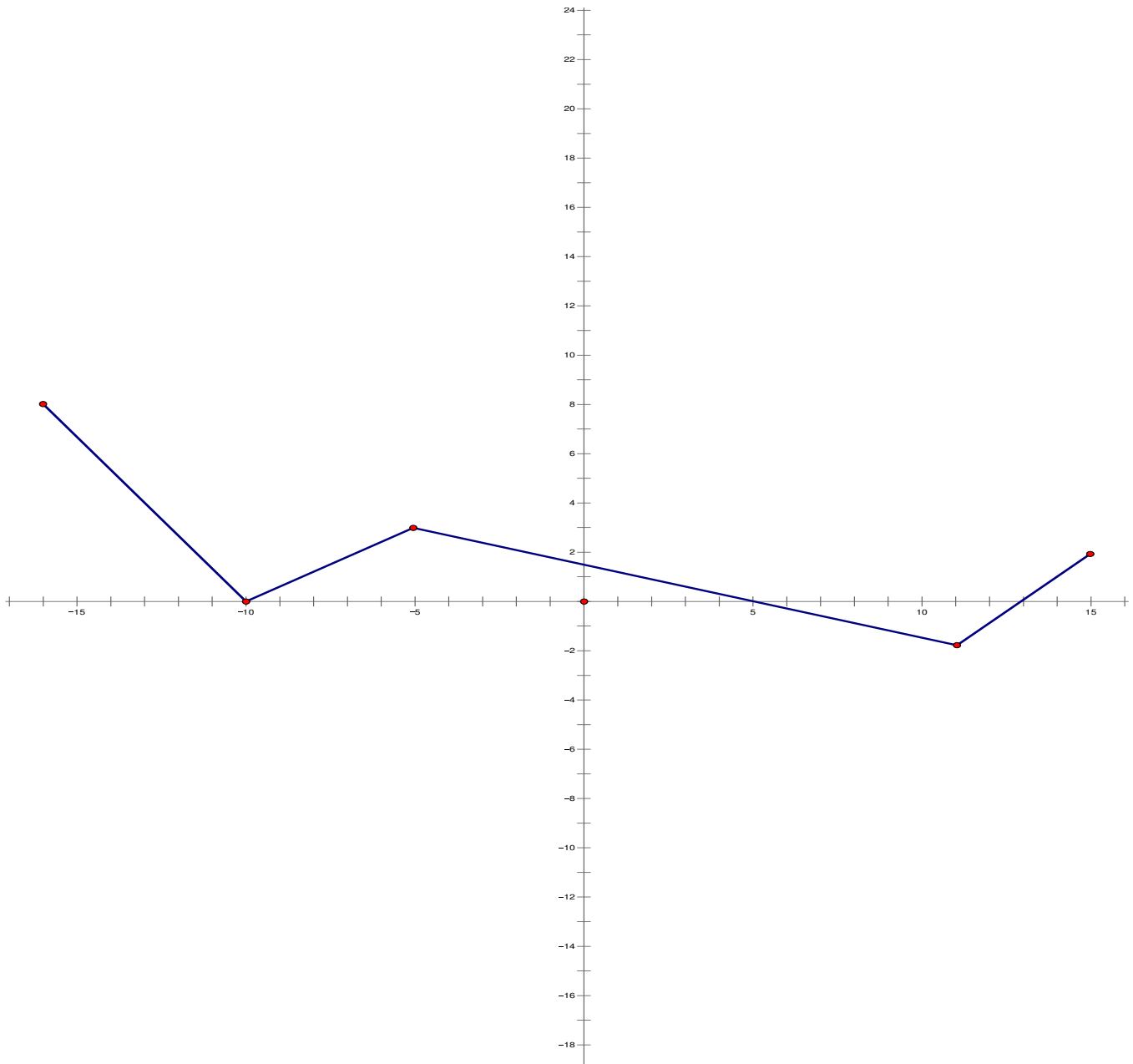
(vi) Given $x^2 + y^2 - xy + xe^y + ye^x - 203421897654 = 0$ Then $dy/dx =$

    a. $\frac{2y - x + e^x}{y - 2x - e^y}$

    b. $\frac{y - 2x - e^y}{x - 2y - e^x}$

    c. $\frac{2x - y + e^y}{2y - x + e^x}$

    d. $\frac{y - 2x - e^y}{2y - x + e^x}$

(vii) $\lim_{x \to 3} \frac{x^2 - 10}{(x - 2)^2} =$

    a. -1

    b. $-\infty$

    c. ∞

    d. DNE (does not exist)

(viii) Given the curve of $f'(x)$ on the interval [-16, 15] (i.e., $-16 \leq x \leq 15$). Then

   a. $f(x)$ is decreasing on the the intervals $(-16, -10) \cup (-5, 11)$

   b. $f(x)$ is decreasing on the interval $(5, 13)$

   c. $f(x)$ is increasing on the intervals $(-10, -5) \cup (11, 15)$

   d. $f(x)$ is increasing on the interval $(-16, 13)$

(ix) Using the curve of $f'(x)$ above. Then

   a. $f(x)$ has a critical value at $x = -10$ but $f(x)$ has neither min. value nor max. value at $x = -10$.

   b. $f(x)$ has a local max. value at $x = -5$ and a local min. value at $x = 11$ and $x = -10$.

   c. $f(x)$ has a local min. value at $x = 5$

   d. $f(x)$ has a local max. value at $x = 11$.

(x) Using the curve of $f'(x)$ above. Then

   a. the curve of $f(x)$ must be concave down on the intervals $(-16, -10) \cup (-5, 11)$.

   b. the curve of $f(x)$ must be concave up on only the interval $(-16, -5)$

   c. the curve of $f(x)$ must be concave up on the intervals $(-16, -5) \cup (5, 15)$

   d. the curve of $f(x)$ must be concave down on the interval $(-10, 11)$

   e. the curve of $f(x)$ must be concave down only on the interval $(-10, 5)$

(xi) Given x, y are two positive REAL numbers such that $xy = 3$ and $x + 12y$ is minimum. Then $x + 12y =$

    a. 37

    b. 15

    c. 12

    d. 6.5

    e. 13

(xii) What is the area of the largest rectangle that can be drawn as in the figure below (note $f(x) = -x^2 + 8x + 11 = 27 - (x-4)^2$)?
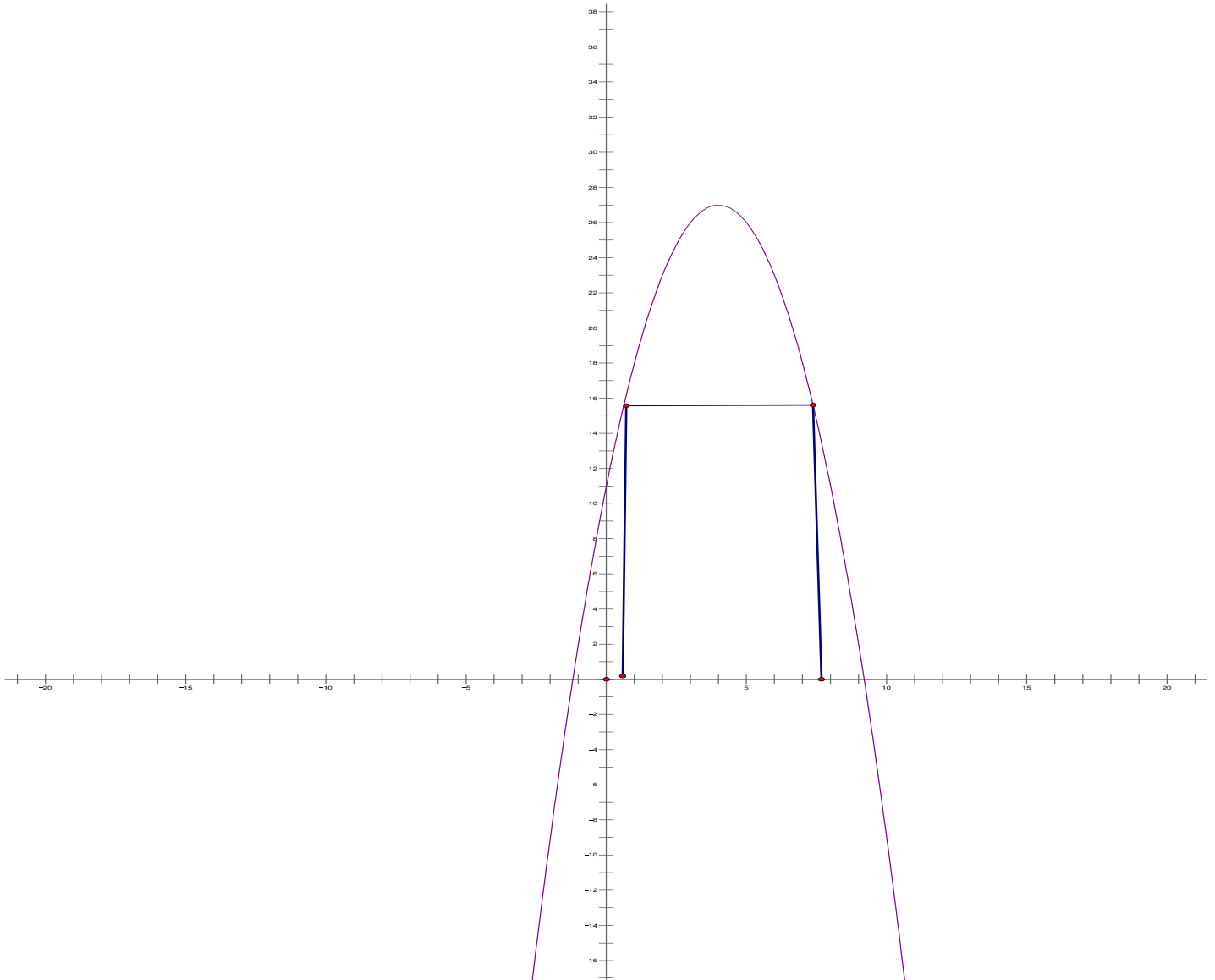
    a. 54

    b. 144

    c. 126

    d. 216

    e. 108

(xiii) the distance between the point $Q = (1, 1, 6)$ and the plane: $3x - 4z - 9 = 0$ is

    a. 6

    b. 2

    c. 0.2

    d. 1.2

(xiv) Given the points $A = (3,4)$ and $B = (8,10)$. What is the point c on the horizontal line $y = 2$ so that $|AC|+|CB|$ is minimum?

    a. $(5\frac{1}{7}, 2)$

    b. $(4, 2)$

    c. $(5\frac{6}{7}, 2)$

    d. $(3, 2)$

    e. $(8, 2)$

    f. $(5.5, 2)$

(xv) Given $f(x) = ln[\frac{(4x-7)^5}{3x-5}]$. Then $f'(2)$

    a. 17

    b. $\frac{20}{3}$

    c. $\frac{4}{3}$

    d. 4

    e. 5

    f. 2

(xvi) $lim_{x \to 2} \frac{e^{(4x-8)}+x^2-5}{x^3+x^2-12} =$

    a. 0.5

    b. $\frac{5}{16}$

    c. 0.25

    d. 0.8

    e. 0

(xvii) Let C be an arbitrary point on the ellipse $\frac{(x+1)^2}{4} + y^2 = 9$, and let c_1, c_2 be the foci of the ellipse. Then $|Cc_1| + |Cc_2| =$

    a. 4

    b. 12

    c. 6

    d. 2

(xviii) Given the parabola $y = 0.1(x-2)^2 - 2$. Then the directrix is

    a. x = 4.5

    b. y = 4.5

    c. y = 2.5

    d. y = - 4.5

    e. y = - 2.5

(xix) Let P be the parabola as in the above question. Given that the point $Q = (12, 8)$ lies on its curve, and C be its focus. Then $|QC| =$

    a. 7.5

    b. 3.5

    c. 5.5

    d. 12.5

    e. 10.5

(xx) The intersection of the plane $x + y = 0$ and the plane $2x - z = 0$ is the following line.

    a. $x = -t, y = t, z = -2t$

    b. $x = -t, y = -t, z = -2t$

    c. $x = t, y = t, z = -2t$

    d. $x = t, y = -t, z = -2t$

(xxi) Given the hyperbola $\frac{(x-1)^2}{9} - \frac{y^2}{16} = 1$. Let c_1, c_2 be the foci of the hyperbola, and C be an arbitrary point on the curve. Then $||Cc_1| - |Cc_2||$

     a. 6

     b. 10

     c. 4

     d. 8

     e. 3

     f. 5

(xxii) One of the following is a foci of the above hyperbola.

     a. $(1, 5)$

     b. $(-1, 5)$

     c. $(6, 0)$

     d. $(1 + \sqrt{7}, 0)$

     e. $(1, \sqrt{7})$

(xxiii) $\int (2x - 7)^8 \, dx =$

     a. $\frac{(2x-7)^9}{18} + c$

     b. $\frac{(2x-7)^9}{4.5} + c$

     c. $\frac{(2x-7)^9}{9} + c$

     d. $\frac{(2x-7)^9}{2} + c$

(xxiv) $\int xe^{(x^2+4)} + 4x - 1 \, dx =$

     a. $0.5e^{(x^2+4)} + 2x^2 - x + c$

     b. $2e^{(x^2+4)} + x^2 - x + c$

     c. $e^{(x^2+4)} + 2x^2 - x + c$

     d. $2e^{(x^2+4)} + 2x^2 - x + c$

(xxv) Given $f'(x) = 4e^{(2x-6)} + 4x + 1$ and $f(3) = 27$. Then $f(x) =$

     a. $4e^{(2x-6)} + 2x^2 + x + 2$

     b. $2e^{(2x-6)} + 2x^2 + x + 4$

     c. $e^{(2x-6)} + 2x^2 + x + 5$
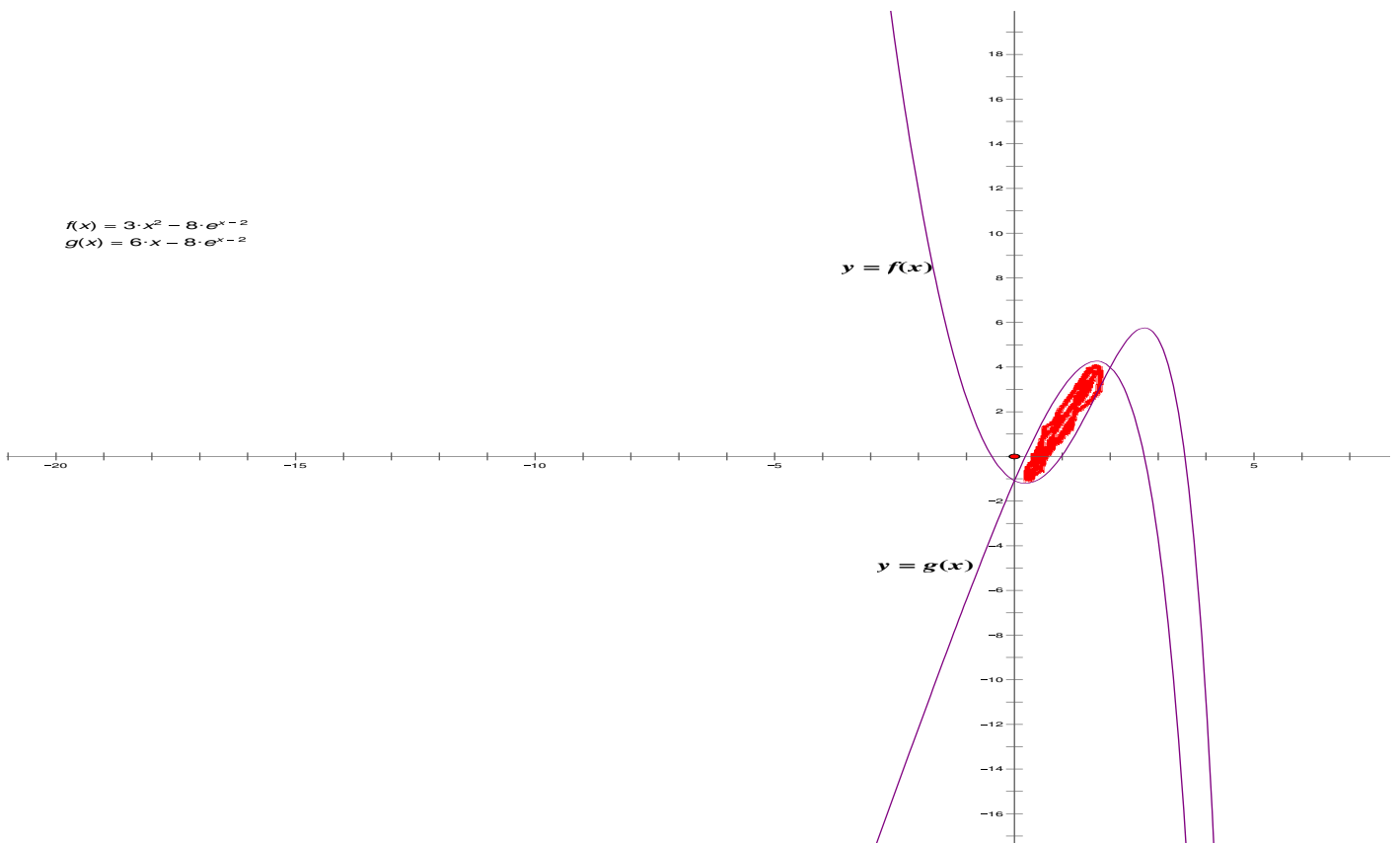
     d. $0.5e^{(2x-6)} + 2x^2 + x + 5.5$

(xxvi) one of the following vectors can be drawn inside the plane : $3x - 2y + z = 4$

     a. $2i + 3j + k$

     b. $3i - 2j + k$

     c. $3i - 2j - k$

     d. $3i - 2j - 13k$

(xxvii) Given the points $(0, 1, 1), (0, \ 2.5 \ 3), (2, 1, 1)$ are vertices of a triangle. The area of the triangle is :

     a. 25

     b. 12.5

     c. 5

     d. 2.5

(xxviii) Find the area of the region bounded by $f(x) = 3x^2 - 8e^{(x-2)}$ and $g(x) = 6x - 8e^{(x-2)}$ (see the region below and notice that the region is between $0 \le x \le 2$)

a. 20

b. 4

c. 36

d. 16

(xxix) One of the following lines lies entirely inside the plane: $x + 2y + 3z - 6 = 0$

a. $x = 1 - 2t, y = 1 + 4t, z = 1 - 2t$

b. $x = 1 + 3t, y = 1 + 2t, z = 1$

c. $x = 1 + t, y = 1 + 2t, z = 1 + 3t$

d. $x = t, y = t, z = 3 - t$

(xxx) $L_1 : x = t, y = 2t, z = 1 + t$, $L_2 : x = 1 + 2s, y = 2 - s, z = 3 + 2s$. Then

a. L_1 intersects L_2 at $(1, 2, 3)$

b. L_1 intersects L_2 at $(1, 1, 2)$

c. L_1 intersects L_2 at $(1, 2, 2)$

d. L_1 does not intersect L_2

(xxxi) Given $(0, 1, 1), (0, 2, 3), (2, 1, 1)$ lie in a plane P. Then an equation of P is

a. $-4(y - 1) - 2(z - 1) = 0$

b. $2x + 4(y - 1) - 2(z - 1) = 0$

c. $4(y - 1) - 2(z - 1) = 0$

d. $4(y - 1) + 2(z - 1) = 0$

e. $-2x - 4(y - 1) - 2(z - 1) = 0$

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